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# When Is Uncertainty About Uncertainty Worth Characterizing?

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In areas of risk assessment ranging from terrorism to health, safety, and the environment, authoritative guidance urges risk analysts to quantify and display their uncertainties about inputs that significantly affect the results of an analysis, including their uncertainties about subjective probabilities of events. Such "uncertainty characterization" is said to be an important part of fully and honestly informing decision makers about the estimates and uncertainties in analyses that support policy recommendations, enabling them to make better decisions. But is it? Characterization of uncertainties about probabilities often carries zero value of information and accomplishes nothing to improve risk-management decisions. Uncertainties about consequence probabilities are not worth characterizing when final actions must be taken based on information available now.

> "But there seemed to be no chance of this, so she began looking at everything about her to pass away the time."
>
> Lewis Carroll, *Alice in Wonderland*

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Quantitative risk analysts are blessed today with a wealth of advice and instruction designed to help them live more useful lives and to communicate their speculations and uncertainties to policy makers with computer-aided clarity. Where previous generations were restrained by the conviction that some uncertainties did not matter (for example, because zero times $X$ is zero, no matter how great the uncertainty about $X$), today's analysts have been liberated and empowered by uncertainty-analysis software.

"Your estimates cannot be more precise than their most uncertain component," they are told (Office of Management and Budget (OMB) 2003). The duty of every good analyst is to characterize uncertainties and to present them to decision makers. Plentiful guidance is available from and for regulatory analysts: "Apply a formal probabilistic analysis of the relevant uncertainties—possibly using simulation models and/or expert judgment as revealed, for example, through Delphi methods. Such a formal analytical approach... is required for rules that exceed the \$1 billion annual threshold. ... You should make a special effort to portray the probabilistic results—in graphs and/or tables—clearly and meaningfully" (ibid). Inspiring words to aspire to... but can we live by them?

Perhaps the desire to tell all can be overdone. Are uncertainties about risks always worth characterizing? *Are uncertainties about these uncertainties useful*?

## Quantifying a Probability

Suppose that your client, a government-agency functionary who is responsible for important risk assessments, asks you (the expert) for your subjective degree of belief (let's call it "$p$") that a particular technology will become available to the general public within three years. If it does, then a distribution over consequences (sometimes called a "risk curve") to our country can be calculated using a standard and well-validated model; if it does not, then this model can be used to produce a *different* distribution over consequences.

After doing whatever it is that made you an expert, you answer, "$p = 0.35$." This is too simple for your client, who reproachfully reminds you that point estimates are for chumps, and that "Monte Carlo and other probabilistic methods—simulating a distribution of the results by randomly drawing from the probability distributions of input variables and repeating the analysis numerous times" are now de rigueur (US Environmental Protection Agency (EPA) 1999). The client shows you copies of uncertainty-analysis policies from other agencies and agency heads, past and present, emphasizing that "First, we must insist on risk calculations being expressed as distributions of estimates and not as magic numbers" (National Research Council (NRC) 1994, p. 160). The client further admonishes you that it is key to distinguish between epistemic and aleatory sources of uncertainty in your estimate.

You look at your "$p = 0.35$" and feel embarrassed. You are not sure what epistemic uncertainty is, and you do not know whether aleatory uncertainty will turn out to matter, but you are pretty certain that your response of 0.35 is going to need some enhancements.

"Can you take a few months," asks your client, "to recognize that, because you are uncertain about its value, the probability you just gave me is in fact a random variable $P$, and therefore please produce a *distribution* for it? Then I can present this to my agency's decision makers with appropriately detailed quantitative support."

You agree to do so (perhaps feeling that life offers too few opportunities to get paid for such deep introspection).

## Elicitation of Uncertainty About a Probability

Specialists in expert elicitation train you on calibration and bias and the best approaches for eliciting subjective distributions. You oblige them and produce $f(p)$, a distribution for $P$. This distribution, not surprisingly, turns out to have an expected value of $E(P) = 0.35$! More importantly, during this time, you realize that a critical factor you subjectively used in coming up with the distribution over $P$ (and the value 0.35 in the first place) was whether a particular chemical process can be stabilized within the next six months.

Pleased with your efforts, your client takes your resulting subjective uncertainty distribution $f(p)$, having expectation of 0.35, and generates plots of it using various graphics programs and a new cluster of supercomputers recently procured to facilitate fuller and more responsible uncertainty characterization in these troubled times. This software displays quintiles of your uncertainty distribution, prepares bar charts with multicolored, nested, subjective confidence intervals, and fits parametric distributions and cubic splines to your results. In addition, it almost automatically adds graphical appendices to your original report (that was a mere e-mail with the unassuming estimate "$p = 0.35$" in its text) to show how rich and deep your reflections about that 0.35 really are.

With the remaining budget—your client's budget this year for expert elicitation and uncertainty analysis seems almost limitless as agencies responsible for addressing issues ranging from climate change to the war on terror realize how impressive uncertainty analyses look—he enhances the graphs to show deciles and other quantiles of your distribution $f(p)$, initiates a research project at a major university to develop resampling methods to better estimate the parameters of the distributions that approximate $f(p)$, and convenes a four-day workshop to discuss the importance of clearly displaying uncertainties about uncertainties about risks.

## Zero Information Value of Uncertainty Characterization

At the workshop, you fall into a somnolent reverie. You realize that all of this well-meant activity has added precisely nothing (value of information (VOI) = 0) to your initial assessment of $p = 0.35$. It seems to be a pure waste of resources from the standpoint of improving or informing decisions.

Why? Because, at the end of this tortuous process, you are still saying neither more nor less than that you judge the probability that a particular technology will become available to the general public within three years to be 0.35. No amount of additional "elicitation" and no artistic rendering of your uncertainty about that number changed it in the slightest or added any value to it. The distribution of your uncertainty about that probability might be interesting from a philosophical point of view, and might even be useful—but *only if* you were to be given additional information about the progress in stabilizing the critical chemical process. That is, the VOI about the chemical process stabilization might be greater than zero. However, you *were not* given this information; indeed, if you *had been* given it, your whole assessment would have changed in response.

The bottom line is that you have been asked for a probability, given the information and insights you have *now* (not what you might have learned had additional information been available). And, as you were told in your introductory probability course, this *probability*—the expected value of a binary indicator for the event—is *simply a number*, *not a distribution*.

## The Brave New World of Uncertainty Characterization

You awaken from these reflections to hear the workshop's plenary speaker—famous for his work at the boundary of neuroeconomics and risk perception—suggesting that "yes," "no," and "maybe" are biologically meaningless concepts, unworthy of use in scientific uncertainty analysis, and that continuous distributions of values (corresponding to dopamine gradients) are essential for biologically realistic risk communication between parts of the brain. You know this will appeal to the many subject matter experts you have met who refuse to commit to such crisp terms as "always" or "never."

You sit up when the speaker shows an exciting film clip of functional magnetic resonance images (fMRI) of the posterior cingulate cortex of your client agency's risk manager while he is being offered reinforcements of (1) fruit juice or (2) video clips of animated 3D uncertainty distributions for risk estimates.

You are a bit disappointed to hear that it was ambiguous which reinforcement, if either, more effectively primed the risk-management brain centers (but the false-color images were terrific!), and you are chagrined to reflect that all you really had to say about that technology becoming available was that you thought it had a 35 percent chance. (Admittedly, the visualization now scrolling across the huge auditorium screen, showing 3D animations of your uncertainty distribution as approximated from different bootstrap samples, and using alternative statistical models and a Bayesian model-averaging postprocessing step, almost convinces even you that there must be more to the story than that.)

## Epilogue

You have met some nice people who clearly know a lot about expert probability elicitation and firmly believe that it will empower better decisions. They did seem a little vague about how, but reassured you that current scholarship holds that "the onus is on the communicators of the probabilistic information to help people find better ways of using the information, in such a manner that respects the users' autonomy, full set of concerns and goals, and cognitive perspective" (Patt and Dessai 2005, p. 437).

You eagerly anticipate working next year with the risk-communication team and its fMRI machine to make sure that your probability assessments are communicated with proper respect for all possible decision-maker concerns, goals, and perspectives.

And you can hardly wait to meet the decision makers who will be funding this uncertainty-about-uncertainty characterization effort for many years to come. Will they like your work? Will they remember to analyze the robustness of their decision with respect to $p$? The client has been effusive in his compliments of the thick and colorful report you helped to produce, and has told you that the decision makers were delighted with the graphics (especially the smooth gradations of colors to suggest mixtures of aleatory and epistemic uncertainties). Will they continue to fund it? You are not sure. But you are comfortable giving it a probability of 0.99.

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